"On the Association of Attributes in Statistics, with Examples from the Material of the Childhood Society, &c." By G. UDNY YULE, formerly Assistant Professor of Applied Mathematics, University College, London. Communicated by KARL PEARSON, F.R.S. Received October 20,—Read December 7, 1899.

(Abstract.)

The paper deals with the theory of association of attributes, *i.e.,* invariable attributes, as opposed to the "correlation" of variables. Two attributes A and B are independent or unassociated if

$$(AB) = (A)(B)/N,$$

(A) being the frequency of the attribute A, (B) that of B, and (AB) the frequency of the pair AB; N being the total number of observations. If this relation do not hold, they are "associated."

Section (I) of the paper is introductory, describing the subject-matter and notation, which is essentially that of Jevons.* Calling a group defined by n attributes ABCD......N an nth-order group, Section (II) deals with the fundamental problem of the number of independent nth order frequencies that can be formed from m attributes; *i.e.,* the number of such frequencies that must be given before the remaining frequencies of the same order can be calculated. Certain extremely curious relations are shown to hold in the special case of "equality of contraries," where all pairs of contrary frequencies (A) (α), (AB) $(\alpha\beta)$, (ABC) $(\alpha\beta\gamma)$ are equal, α being the contrary of A—*i.e.* not A—and so on.

Section (III) proceeds to the theory of association proper. The function

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

is proposed as a "coefficient of association." It is zero when the attributes are independent, $+1$ when all A's are B or all B's are A, and -1 when all A's are β or all β's are A, and thus measures the approach towards "perfect association" in the same sort of way as the correlation coefficient measures the approach towards perfect correlation. The connection between correlation and association is touched upon, and it is pointed out that one may form "partial" coefficients of association (by limiting the extent of the universe of

* "On a General System of Numerically Definite Reasoning," 'Manchester Lit. and Phil. Soc.,' 1870, and "Pure Logic and other minor works" p. 173.

discourse) roughly corresponding in their uses to partial coefficients of correlation.

In Section (IV), the values of the Probable Errors, and the correlations of the errors in the chief constants, are obtained. The probable error of Q is

$$0\cdot6745\,\frac{1-Q^2}{2}\sqrt{\frac{1}{(AB)}+\frac{1}{(A\beta)}+\frac{1}{(\alpha B)}+\frac{1}{(\alpha\beta)}}\,.$$

In Section (V), a series of miscellaneous illustrations are given (association of smallpox attack rate and non-vaccination; association between temper of husband and wife, inheritance of artistic faculty &c., from Mr. Francis Galton's 'Natural Inheritance'; association between vigour of offspring and crossing of parentage in plants from Darwin's 'Cross and Self-fertilisation').

In Section (VI), the "Association of defects in children and adults," is treated more at length as an example of the methods advocated, the material being drawn mainly from the Report of the Committee on the Scientific Study of Childhood. It is shown that the association coefficient is almost uniformly higher for women than men, and for children than adults. This last effect is however a mixed one, due partly to selection, partly to change in the individual, and the material available does not enable us to separate the partial effects. These two laws of association appear to correspond to similar ones for correlation; women being more highly correlated than men, and children than adults.

"Data for the Problem of Evolution in Man. III.—On the Magnitude of certain Coefficients of Correlation in Man, &c." By KARL PEARSON, F.R.S., University College, London. Received November 20,—Read December 7, 1899.

1. This paper contains a number of data bearing on the correlation of characters, &c., in man which have been worked out by my collaborators during the last few years, and several of which seem of considerable importance for problems relating to the evolution of man. In each case the data were procured or reduced with a view to answering some problem which had directly arisen during our inquiries as to the action of natural selection on man. Questions as to the alteration of correlation with growth or the influence of homogamy on fertility demand definite answers before the general theory of the influence of natural selection on a growing and reproductive population can be effectively developed.